A Measurement of $g$

Listening to Falling Balls

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A ball placed on the edge of a horizontal table is hit with a pendulum and thrown over the floor. The fall time is measured by recording the sounds produced by the collisions with the pendulum and the floor. If the height of the ball with respect to the floor is known, one can determine the acceleration of gravity.

The measurement of the acceleration of gravity, $g$, is a usual experiment in the introductory physics laboratory. Most of the traditional methods are based on the measurement of the time for the free fall of a body (usually a spherical ball) through a known distance. In order to neglect air drag effects, small distances (less than 1 m) are convenient and, as a consequence, the fall times are rather short (less than 0.5 s). Therefore, manually operated chronometers are inadequate. In that case, optical position sensors or electrical contacts are usually employed as start/stop switches for a commercial electronic counter timer or a timing circuit in the millisecond range.\(^1\-6\)

An alternative way to measure times with millisecond resolution is by digitally recording audio signals with large enough sampling frequency. This technique was employed some years ago\(^7\,8\) to determine the coefficient of restitution of a bouncing ball. The method was recently\(^9\,10\) improved by using modern personal computers (PC) equipped with a sound card and software programs for sound recording.

Here we propose a method to determine $g$ based on the observation of a falling ball in two dimensions (parabolic throw) instead of the usually employed free-fall (one-dimensional) methods. The method consists of throwing, with the help of a pendulum, a stainless steel ball from a known height onto the floor and measuring the fall time by recording the sounds produced by the collisions with the pendulum and the floor.

Figure 1 shows the experimental setup. A stainless steel ball (b) of known radius $R_b$ is placed just on the edge of a horizontal table. By using a pendulum as a pushing mechanism, this ball is thrown onto the floor. The pendulum (p) consists of a stainless steel ball of known radius $R_p$ slightly smaller than $R_b$. The acceleration of gravity $g$ can be determined by measuring the time $t_f$ for the ball to fall a vertical distance $H$,\(^\)\(^1\)

$$g = \frac{2H}{t_f^2}. \quad (1)$$

In order to ensure the reproducibility of the experiment, it is essential to throw the falling ball always in the same manner. With this aim, the following considerations are necessary:

(1) The ball must start from rest with its center vertically aligned with the edge of the table. A piece of adhesive tape placed parallel to the edge helps for this alignment.

(2) At the moment of the collision, the center of the falling ball and the pendulum should be horizontally aligned. This alignment guarantees that the motion of the ball starts horizontally from the table.

(3) A thread is joined to the pendulum with Sellotape and clamped with a clothespin placed in the verti-
cal bar supporting the pendulum. A mark in the thread helps with always throwing the pendulum from the same position. Before release, the distance between the pendulum and the falling ball should be large enough to avoid any rotational motion of the ball after collision.

The experiment is run as follows. The height $H$ is measured with a 1-m steel rule with 0.5-mm divisions. The falling ball is placed in its starting point according to the scheme of Fig. 1. The pendulum is suspended in the air by clamping the thread with the clothespin. After opening the clothespin, the pendulum moves and collides with ball $b$ throwing it onto the floor, where the impact point is marked (for example, by using carbon paper). A standard microphone connected to a PC is then suspended in the air just above the impact point and equidistant from the starting point of the falling ball in order to cancel the time delays due to the limiting speed of sound. Next, one repeats the experiment and records the sound of impact. The PC is equipped with a sound card and a software program for sound recording. In our experiment, the sampling frequency was 44.1 kHz, corresponding to a time resolution of 23 $\mu$s. The sound file recorded in a typical experiment is plotted in Fig. 2. Two sound pulses produced by the collision between balls and the impact of the falling ball with the floor are clearly recognized. Using the zoom tool of the program, one can accurately locate the pulses and easily measure the fall time $t_f$ (time interval between pulses).

The experiment has been done with a stainless steel ball with radius $R_b = 1.25$ cm, placed at height $H = 78.175 \pm 0.025$ cm. A stainless steel ball with radius $R_p = 1.15$ cm was used with the pendulum of length $L = 55$ cm. This ball was always released from a height $h_1 = 2.12$ cm (see Fig. 1). We have carried out 20 throws, obtaining fall times $t_f$ between 399.800 ms and 400.050 ms. The mean value was 399.91 ms with a standard deviation of 0.09 ms. By using Eq. (1), with this mean value one obtains an acceleration of gravity of $977.6 \pm 0.5$ cm/s$^2$. The value of $g$ in Salamanca, provided by the Spanish National Geographic Institute, is $g = 980.04662$ cm/s$^2$ at an altitude of 805 m. Comparison of the experimental value with our measure shows an accuracy of 0.25%.

An improvement of the precision and accuracy of the experiment can be obtained by analyzing and evaluating the principal sources of error, mainly, the initial placement of the falling ball, the air buoyancy and drag effects, and a more accurate measurement of the fall height. In particular, we note that including buoyancy and drag effects would add about 0.07% to the result for $g$.

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References
11. Software for recording and editing sounds can be found on the World Wide Web. An excellent freeware software is provided by “Audacity” (http://www.audacity.sourceforge.net/).

PACS code: Karl please supply

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